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Experimental Verification of a Mathematical Model for Quiescent Settling of a Flocculating Slurry

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Abstract

Quiescent settling experiments were carried out with ferric hydroxide flocs to test a previously proposed theoretical model for settling. The model for orthokinetic flocculation includes a catch-up mechanism associated with the different fall velocities of particles of different sizes, and a turbulence mechanism resulting from the eddies caused by the dissipation of energy by the falling particles. Adjustment of parameters to yield settling velocities in agreement with experimental results at settleable solids volume fraction (SSVF) of 0.03 and 0.20 yielded plots in good agreement with the experimental data over the entire range of SSVF.

MODEL THEORY

The major objective of this work was to experimentally verify a theoretical model for settling which we had proposed earlier (*1–4*). Much of the relevant theory is reviewed in the first of these papers. Other work bearing closely on our approach includes that of S. Chang's dissertation on clarifier operation (*5*) and an extensive report on orthokinetic flocculation by Argaman and Kaufman (*6*).

Our model for othokinetic flocculation is based on:

(1) A catch-up mechanism associated with different terminal velocities of particles of different sizes

(2) A mechanism based on the eddies and turbulence generated in the liquid by the dissipation of energy by the falling particles

This mechanism is related to the work of Camp and Stein (*7*) on the effect of turbulence in orthokinetic flocculation. We herein extend our previous model to include the size-dependence of the floc particle density discussed by Vold (*8*). We then discuss the choice of floc parameters to permit the calculation of settling velocity as a function of settleable solids volume fraction (SSVF) for ferric hydroxide flocs. We develop the theory for *upflow sludge blanket clarifiers*. Quiescent settling constitutes a special case for which the influent and sludge wasting flow rates are zero and the cylindrical clarifier is initially filled with a slurry of uniform composition.

We assume that composite particles aggregate from unit elementary particles or smaller composite particles under the influences of the catch-up and turbulence mechanisms; disrupt under the influence of viscous drag forces; and move up or down under the influences of gravity, viscous drag, and bulk flow of the liquid in the clarifier.

Our continuity equation is then

$$\frac{\partial c_n}{\partial t}(x, t) = -\frac{1}{A(x)}\frac{\partial}{\partial x}(Av_n' c_n) + \frac{1}{A(x)}\frac{\partial}{\partial x}\left(AD_n\frac{\partial c_n}{\partial x}\right) + F_n(c(x, t)),$$
$$n = 1, 2, \ldots, N \qquad (1)$$

The catch-up mechanism and floc disruption process contribute the following terms to F_n (see Ref. *1*):

$$F_n = \sum_{j=1}^{(\frac{n}{2})} c_j c_{n-j}|v_j - v_{n-j}|\pi(r_j + r_{n-j})^2 - \sum_{j=1}^{N-n} c_j c_n|v_j - v_n|\pi(r_j + r_n)^2$$
$$+ \sum_{j=n+1}^{N} k_{n,j-n}^{j} c_j(1 + \delta_{n,j-n}) - \sum_{j=1}^{(\frac{n}{2})} k_{j,n-j}^{n} c_j \qquad (2)$$

where $\delta_{ij} = 0$, if $i \neq j$; $\delta_{ij} = 1.0$, if $i = j$.

$$\left(\frac{n}{2}\right) = \text{largest integer} < \frac{n}{2}$$

$$k_{n,i-n}^{j} = \frac{\kappa j j!}{n!(j-n)!} \cdot \frac{\left(\frac{n}{2}\right)!\left(N - \left(\frac{n}{2}\right)\right)!}{NN!} \qquad (3)$$

$$\kappa = \text{proportionality constant (sec}^{-1}\text{)}$$

We use Vand's formula (*9*) for calculating viscosity as a function of volume fraction solids C,

$$\eta = \eta_0 \exp\left(\frac{2.5C + 2.7C^2}{1 - 0.609C}\right) \tag{4}$$

where η_0 = viscosity of the pure liquid (poise)

The slurry density is given by

$$\rho_{sl} = \rho_s C' + \rho_l(1 - C') \tag{5}$$

where

$$C' = \sum_{n=1}^{N} c_n n v_1 \tag{6}$$

ρ_s = density of an elementary particle (g/ml)

ρ_l = density of liquid (g/ml)

We assume Vold's formula (*8*), but not necessarily her value of p, for the volume of an n-particle:

$$V_n = V_1 n^p \tag{7}$$

With this, the density of an n-particle is

$$\rho_n = \frac{\rho_s + \rho_l(n^{p-1} - 1)}{n^{p-1}} \tag{8}$$

and the difference in density between an n-particle and the surrounding slurry is given by

$$\Delta\rho_n = \rho_n - \rho_{sl} = (\rho_s - \rho_l)\frac{1 - C'n^{p-1}}{n^{p-1}} \tag{9}$$

The radius of an n-particle we take as

$$r_n = \left(\frac{3V_n}{4\pi}\right)^{1/3} \tag{10}$$

The velocity of an n-particle relative to the surrounding liquid is given over a rather broad range of Reynolds numbers by (*10*)

$$-v_n \equiv u_n = \frac{2g\Delta\rho_n r_n{}^2}{9\eta\left[1 + \frac{1}{4}\left(\frac{\rho_{sl} r_n u_n}{2\eta}\right)^{1/2} + 0.34\left(\frac{\rho_{sl} r_n u_n}{12\eta}\right)\right]} \tag{11}$$

where g = gravitational constant (980 cm/sec^2).

The velocity of an n-particle relative to the laboratory is then given by

$$v_n' = v_n - \sum_{k=1}^{N} v_k c_k V_k + \frac{Q_{\text{feed}}}{A(x)} \tag{12}$$

below the sludge wasting plane, and by

$$v_n' = v_n - \sum_{k=1}^{N} v_k c_k V_k + \frac{Q_{\text{feed}} - Q_{\text{waste}}}{A(x)} \tag{13}$$

above this plane.

The turbulence mechanism contributes the following terms to F_n in Eq. (1) (see Ref. *4*):

$$\begin{aligned} F_n'' = \sum_{j=1}^{(\frac{n}{2})} \frac{4}{3}\alpha G(r_j + r_{n-j})^3 c_j c_{n-j}\left(1 - \frac{\delta_{j,\, n-j}}{2}\right) \\ - \sum_{j=1}^{N-n} \frac{4}{3}\alpha G(r_j + r_n)^3 c_j c_n \end{aligned} \tag{14}$$

$$G = \left(\frac{\varepsilon}{\eta}\right)^{1/2} \qquad \text{(see Ref. 7)} \tag{15}$$

$$\varepsilon = -g(\rho_s - \rho_l)\left(\sum_{n}^{N} v_n c_n n V_1\right)\left(1 - \sum_{n}^{N} c_n n V_1\right) \tag{16}$$

where ε is the power dissipation per unit volume of the falling particles and G is the root-mean-square velocity gradient resulting from this power dissipation. α is a scalar constant of less than unity which we somewhat arbitrarily assign a value of 0.5; see Ref. *4*.

We have included effective axial diffusion constant D_n in Eq. (1) for the sake of generality. Assignments of numerical values to these parameters would be difficult. Axial dispersion effects are probably most conveniently handled in computation by the choice of the thickness Δx of the slabs into which the settling column is partitioned in order to integrate Eq. (1) forward in time. We therefore set $D_n = 0$ in our calculations without further ado. This matter is discussed in more detail in connection with activated carbon columns in Ref. *11*.

We convert Eq. (1) into a set of coupled ordinary differential equations by methods described earlier (*1*, *3*) and integrate by the same predictor-corrector method we used previously. The boundary conditions at $x = 0$ (bottom), $x = l - \varepsilon$, $x = l + \varepsilon$ (immediately below and immediately above the sludge wasting plane), and at $x = L$ (top of clarifier) have been discussed earlier (*3*) and are unchanged here.

EXPERIMENTAL WORK

Our objective for the experimental work was to find the settling velocities of ferric hydroxide suspensions of various concentrations. The ferric hydroxide precipitate used in the settling tests was made by diluting a solution of industrial ferric sulfate in tap water. The iron content of the diluted solution was approximately 100 mg/l. Additional precipitation was forced by raising the pH with either $Ca(OH)_2$ or NaOH. The floc produced from this procedure averaged about 190 mg/l nonfilterable suspended solids. We determined the settleable solids volume fraction (SSVF) by allowing 1 liter of the slurry to remain undisturbed in a graduated cylinder for 24 hr. We calculated the SSVF from the volume occupied by the settled precipitate as a fraction of the initial slurry volume. Our results showed the initial slurry to have a SSVF of 0.02. This SSVF was increased by allowing large volumes of the original slurry to settle to a volume of 1 liter and discarding the supernatant.

We measured the settling velocity of the slurries by the procedure which is used with wastewater sludges that are to be treated in gravity thickeners (*12*). We mixed the concentrated slurries and poured them into transparent 1-liter cylinders. The mixing disrupted the floc so that the slurry was of a uniform, creamy consistency. The floc particles coagulated as they settled, giving the slurry a rough, curdled appearance. As the slurry settled, we recorded the height of the interface at regular time intervals. The settling velocity was taken to be the slope of the straight-line portion of the plot of the interface height versus time. While they settled, the slurries were stirred by vertical shafts rotating at 6 rph to reduce the effects of the small diameter vessel on the settling process.

The settling tests were run on a series of slurries adjusted to pH 6 and to pH 10 with $Ca(OH)_2$. Slurries adjusted to both pH's with NaOH also were used. A fifth series of tests was run with the addition of 5 mg/l Nalco 609 cationic polyelectrolyte to a slurry precipitated with NaOH at pH 6. We varied the SSVF in each type of slurry from about 0.03 to 0.30. Figure 1 shows the results of a typical series of runs. Precipitates of less than 0.03 SSVF did not settle with a distinct interface. All series showed the expected decrease in settling velocity with increase in SSVF. When the SSVF was increased to 0.2, the settling velocity was greatly reduced. Further increases in concentration only slightly reduced the settling velocity. The slurry with the polymer settled noticeably faster than the other slurries at low concentrations.

The use of lime as opposed to NaOH produced very little difference in

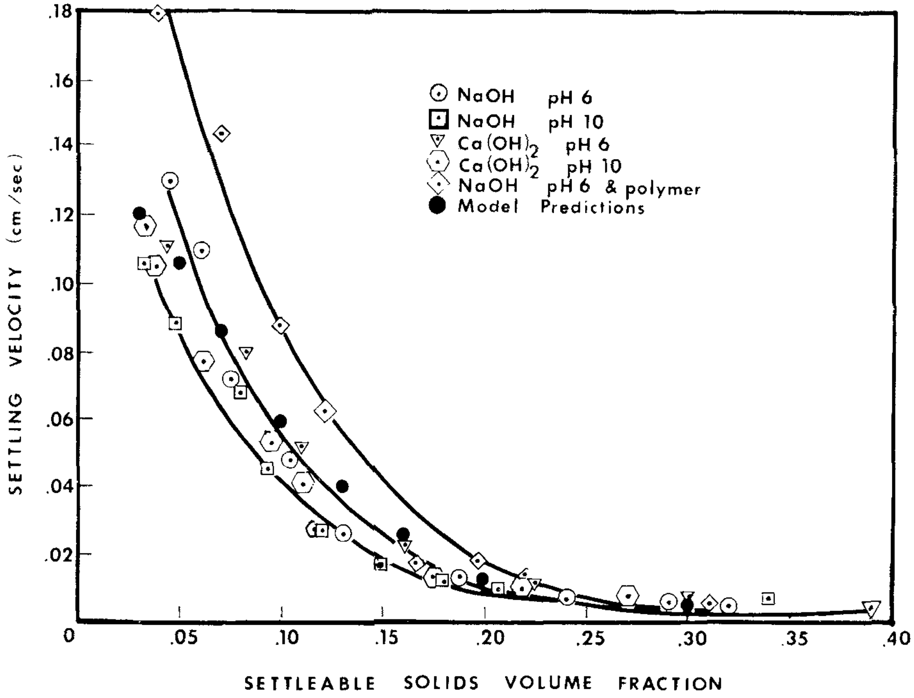

Fig. 1. Settling velocity dependence on settleable solids concentration.

the settling velocity in SSVF relationship. Both types of slurries at low SSVF settled somewhat faster at pH 10 than at pH 6. Lime flocs are often reported as producing a heavier, faster settling sludge than NaOH flocs. Our work shows that iron precipitates of the same SSVF and the same pH settle at practically the same velocity, independent of the neutralizing agent used. However, lime treatment of a given waste may produce a heavier floc as measured by gravimetric suspended solids. For slurries adjusted to pH 6 by NaOH or lime or adjusted to pH 10 by NaOH, we found that 1000 mg/l suspended solids represented a SSVF of 0.10 with a settling velocity of about 0.05 cm/sec. However, for slurries adjusted to pH 10 by lime, 1000 mg/l represented a SSVF of 0.05 with a settling velocity of about 0.1 cm/sec. Thus each concentration measured in mg/l was associated with two settling velocities. A plot of settling velocity versus suspended solids resulted in two curves, one for lime at pH 10 and a second for the other slurries. Therefore the relationship between settling velocity and suspended solids in mg/l can depend on the type and amount of precipitant used in the slurry.

We caution that ferric hydroxide floc formed under conditions different from those described here may settle at quite different rates. We have some evidence that newly formed floc settles as much as 10 times faster than floc that has been disrupted and reformed. It is not safe to assume that floc formed in a flow-through system will settle at the same rate at which floc of equivalent SSVF settles in a graduated cylinder.

NUMERICAL VALUES OF MODEL PARAMETERS

We next explore the assignment of numerical values to the parameters which appear in the theory. Influent and sludge waste flow rates and the geometry of the clarifier design are under our control. The parameters describing the floc, however, must be estimated from experimental measurements. It is desirable to keep these experimental measurements as simple as is consistent with the matching of the theory with the experimental performance of the clarifier. We proceed as follows.

The solids density we estimated by observing that the settled volume of solids obtained from 1 liter of slurry containing 1000 mg/l of Fe(III) is 140 ml. We assume that these solids consist of goethite [FeO(OH), density 4.28 g/cm^3] and trapped water. The mass and volume of the goethite which contains 1000 mg of Fe(III) are 1.5910 g and 0.3717 ml, respectively. The mass of 140 ml of settled solids is then given by

$$1.5910\ \mathrm{g} + (140 - 0.3717)\ \mathrm{ml}\left(0.99923\,\frac{\mathrm{m}}{\mathrm{ml}}\right) = 141.112\ \mathrm{g}$$

at 20°C. Thus the density of the settled solids is 1.0079 g/ml. We note that this calculation is to be regarded as merely a rough estimate, indicating that the value of $\rho_s - \rho_l$ is probably in the range of 0.006 to 0.012 g/ml. The viscosity of water at 20°C we took as 0.0100 P.

The value of N, the number of elementary particles in the largest composite particle permitted, we chose as small as reasonably possible to keep the computer cost reasonable. (Computer running time varies as N^3.) We set $N = 4$ in the work reported here. We assumed that the initial concentration distribution of floc particle sizes is proportional to n^{-2}, where n is the number of elementary particles in an n-particle. We note that our previous work indicates that the results are very insensitive to this initial distribution due to the rapid rate of flocculation. We took k, the scale factor governing the rate of floc breakup, equal to 0.01 sec^{-1}

(with $N = 4$, our results are rather insensitive to the value of this parameter). We chose for r_1, the effective radius of an elementary particle, a value of 0.02 cm, which with our other parameters yields composite floc particles with maximum diameter of about a millimeter, in rough agreement with observed sizes.

We then selected values of $\rho_s - \rho_l$ and Vold's parameter p (see Eq. 7 and Ref. 8) to yield calculated settling velocities in agreement with observed settling velocities at a SSVF of 0.03 and 0.20 near the upper and lower limits of the range covered experimentally. At higher values of the SSVF the calculations indicated that the top boundary of the settling floc in the column was sharply defined (see Fig. 2); at lower concentrations the boundary was less sharp, as seen in Fig. 3. This fuzziness of the boundary at low SSVF was also observed experimentally. The position of

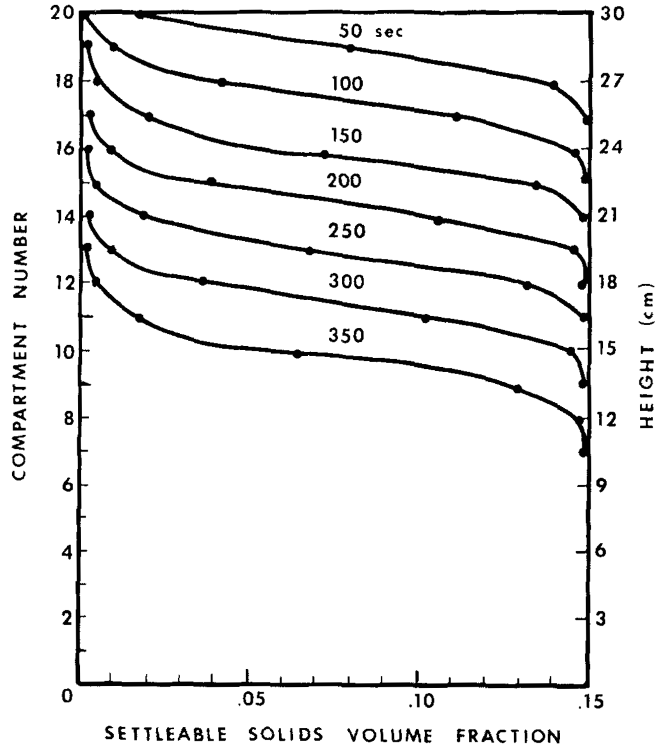

FIG. 2. Quiescent settling concentration profile, SSVF = 0.15.

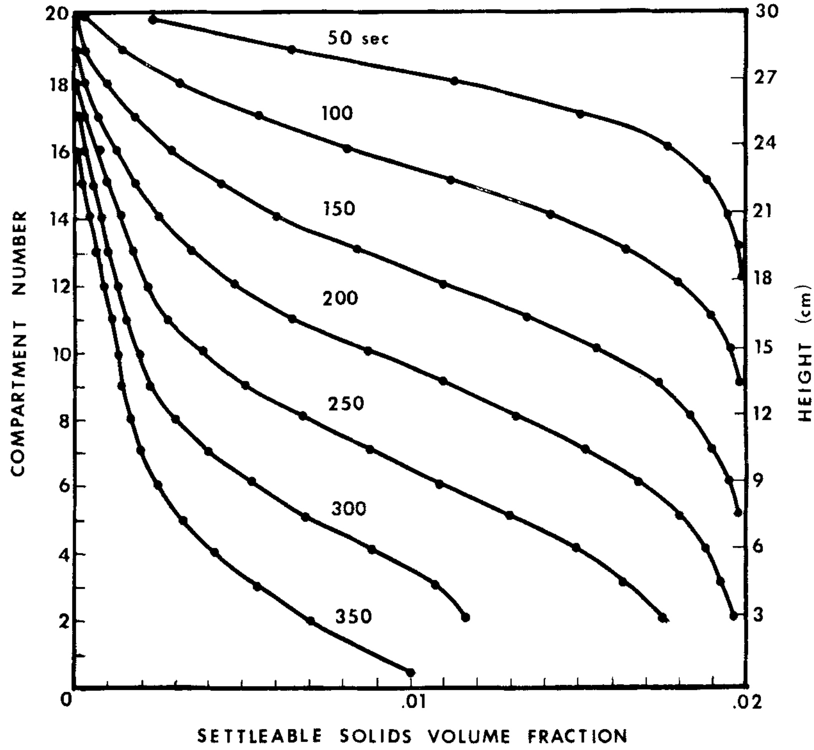

FIG. 3. Quiescent settling concentration profile, SSVF = 0.02.

the point at which the SSVF had decreased to one-half its initial value was determined graphically at various times during the course of the settling, and the settling velocity was calculated from these data.

RESULTS AND CONCLUSIONS

We found that $\rho_s - \rho_l = 0.010$ g/ml and Vold's parameter $p = 1.80$ yielded calculated settling velocities in agreement with our experimental results at an SSVF of 0.03 and 0.20. These values o $\rho_s - \rho_l$ and p were then used to calculate settling velocities of the floc over the SSVF range 0.03 to 0.30. The results are compared with the experimental data in Fig. 1. We note that the range of SSVF covers a factor of 10, that the range of settling velocity is from 0.005 to approximately 0.12 cm/sec, and that the plots show a good deal of curvature. Our theoretical model is able to reproduce the experimental results almost to within the experimental uncertainty resulting from the use of different precipitants [$Ca(OH)_2$ and NaOH] and pH's (6 to 10).

There is a marked enhancement of settling velocity at low SSVF on the addition of polymer. The change is such as can easily be duplicated by the theoretical model by decreasing p and/or increasing $\rho_s - \rho_l$. Both of these changes are consistent with the formation of a somewhat more readily coagulated, tightly bound, and compact floc, which is the effect which the addition of polymer is expected to produce. We emphasize that the addition of polyelectrolyte filter aids changes the floc characteristics and necessitates the use of numerical values of the floc parameters different from those which describe the floc in the absence of polymer.

SYMBOLS

$A(x)$	cross-sectional area of clarifier at a height x above the bottom (cm^2)
C	volume fraction solids, $= \sum_{n-1}^{N} c_n(x, t)V_n$
c_n	number density of n-particles at (x, t)
$\mathbf{c}(x, t)$	$c_1, c_2, \ldots, c_N$
D_n	effective axial diffusion constant of an n-particle at (x, t) (cm^2/sec)
F_n	flocculation and disruption rates for n-particles per unit volume ($cm^{-3}sec^{-1}$)
l	height of sludge wasting plane (cm)
L	height of clarifier (cm)
N	number of elementary particles in the largest composite particle permitted
Q_{feed}	volumetric feed rate (cm^3/sec)
Q_{waste}	volumetric sludge waste rate (cm^3/sec)
t	time (sec)
v_n	velocity of an n-particle relative to the surrounding liquid (cm/sec)
v_n'	velocity of an n-particle in the laboratory frame of reference at (x, t) (cm/sec)
v''	velocity of liquid at (x, t) relative to the laboratory (cm/sec)
V_n	volume of an n-particle (cm^3)
V_1	volume of an elementary particle (cm^3)
x	distance from the bottom of the clarifier (cm)

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